You Cannot Count Back from Infinity: An Infinitesimal Argument against a Past Eternal Universe

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Introduction

The claim 'you cannot count back from infinity' is considered by many classical theists to constitute a *prima facie* case against a past eternal or temporally infinite universe (though many neoclassical or process theists would demur; Dombrowski, 2007). It is often used to bolster formal arguments of the cosmological flavour (such as the Kalām Cosmological Argument; e.g., Craig, 1979; 2013; Craig & Sinclair, 2009) which attempt to prove the necessary existence of a first cause for the universe:

P1: Everything that begins to exist has a cause
P2: The universe began to exist
C1: Therefore: the universe has a cause

The deduction made is then commonly employed to further argue, albeit far less convincingly, for a first cause that is synonymous with a personal agent, invariably characterised as the Abrahamic-style God of Christianity and Islam (e.g., Craig and Sinclair, 2009):

C2: Therefore: If the universe has a cause, then an uncaused personal creator of the universe must exist
C3: Therefore: An uncaused, personal Creator of the universe who is beginningless, changeless, immaterial, timeless, spaceless and enormously powerful and benevolent does exist

This essay is not concerned *per se* with the legitimacy of philosophical cosmological/first cause arguments (including the Kalām) nor their extrapolation to an agentic, personal God (though the evidence detailed here does go some way to counter P2 above). Nor does it discuss the relative merits of eternal or finite cosmological models from physics. Rather, the aim is to demonstrate that the specific claim made against a temporally infinite universe, i.e., 'you cannot count back from infinity', is spurious, being predicated on a number of misrepresentations and misunderstandings of physics, philosophy and mathematics, particularly so the latter. Indeed, it is no exaggeration to consider the low level of mathematical understanding exhibited by those who use the claim 'you cannot count back from infinity' to be on a par with the notoriously facile question endured by evolutionary biologists, 'if we evolved from monkeys why there are still monkeys?' William Lane Craig, arguably the best known and most influential Christian apologist of his generation, is a prominent proponent of the mathematical claim. Therefore, his arguments and claims will be discussed liberally.

Defining 'Infinity'

Three distinct kinds of infinity are commonly assumed by claimants. The first, simply 'infinity' (often labelled with the now outdated term 'actual'), is a well-recognised mathematical concept referring to a set without limit or bound which contains proper subsets whose members or elements can be paired up in a one-to-one correspondence with the elements of the complete set according to Cantor's Principle of Correspondence (see e.g., Hayden & Kennison, 1968; Hinman, 2005). Because infinite sets can differ in size or more properly, their 'cardinality', there exists a hierarchy of possible infinite sets based on their cardinality or transfinite number. This is easily demonstrated. The infinite set with the lowest cardinality is the set of all natural numbers. We are able to pair all of its members with any proper subset of natural numbers, such as the even numbers, the perfect squares, or say, multiples of 9: e.g., 1-9, 2-18, 3-27; *ad infinitum*. However, the set of all real numbers is obviously more numerous than the set of all natural numbers (there being an infinite number of real numbers between any two natural numbers). Thus the set of all real numbers cannot be matched in a one-to-one correspondence with the set of all natural numbers. In addition, there is an infinite set of both rational and irrational numbers between any two rational and irrational numbers. A 'power' set can be constructed from the possible combinations of subsets within any given infinite set. If a set has n elements, then its power set contains 2^n elements. For example, an infinite set comprised of subsets {a, b, c} consists of eight (i.e., 2^3) power sets; {a, b, c}, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}, as well as an empty set { }. The power set of any given infinite set always contains more elements than the full set. This relationship differentiates infinite sets from finite sets as Euclid's Maxim states that the number of elements in a finite set must be larger than the number of elements in any of its subsets.

Set theory is governed by the Zermelo-Fraenkel axioms (Hayden & Kennison, 1968 and Hinman, 2005 offer good introductions). Accepting the axioms (as the majority of mathematicians do; the axioms only work in conjunction with first order predicate logic so there is no credible alternative) implies acceptance of the existence of at least one infinite set and thus gives us no logical or mathematical reason to rule out an infinite temporal universe or an infinite temporal regression (simply denying the validity of infinite set theory is inadequate; formulating a superior mathematical theory would be necessary). Thus set theory is considered to be a central pillar of mathematics and one of its most notable contemporary proponents has made the following bold prediction (Woodin, 2011a):

"*In the next 10,000 years there will be no discovery of an inconsistency in these theories..... In fact, I make the stronger prediction: There will be no discovery ever of an inconsistency in these theories.*"

The second kind of infinity, a 'potential' infinity, is a philosophical concept dating back to Aristotle (who only had experience of finite numbers). Not generally recognised by contemporary mathematicians, the concept has no application whatsoever in contemporary set theory (Hayden & Kennison, 1968; Lindsay, 2013). Indeed, it has never had any such application. Georg Cantor, the originator of set theory, referred to potential infinity as a "*uneigentlich-unendliches*" or "*improper infinity*" (1883) in some of his earliest work. Nevertheless, a potential infinity is usually defined as an iterative process, procedure or algorithm which will continue endlessly unless some stop or halt condition is specified beforehand. Prominent Christian apologist Kirk Durston (unpublished manuscript), defines it as:

"*.....a procedure that gets closer and closer to, but never quite reaches, an infinite end.*"

Potential infinities are often exemplified by the notion of a sequential count (such as a count of natural numbers: 0, 1, 2, 3.....) but this is mistaken for a number of reasons. First, the term 'potential' implies that something is possible should necessary conditions permit. But in the case of potential infinities no such necessary conditions can exist. The definition of a potential infinity is contradictory because it never involves any infinite goal ("*never quite reaches*"). Second, even if Durston's definition is accepted as is, potential infinities can never be verified since there would be no reliable way of establishing that such a process or procedure will ever terminate until it actually does so. Third, there is no point in a sequential count at which a natural finite number can have a natural infinite number as its successor, so reaching infinity is never a potential outcome in this example; the sequence is and will remain a finite set. While the set of all possible natural numbers (and subsets such as even, odd, or prime numbers etc.) comprise an infinite set, there is no natural (or real, or rational) number that acts as a predecessor to infinity *via* any iteration such as $n + 1$. All we will ever have is an algorithm or procedure with an inexhaustible number of calculations of $n + 1$. The same obviously applies to multiplication; recursively doubling or tripling natural numbers, for example, will only ever achieve a perfectly countable finite number that alone, or in conjunction with its predecessors, would never satisfy the properties required to be elements within an infinite set. So, crucially important to the present discussion is the understanding that we can neither count our way to infinity from zero (or any other starting point) nor equate with or impose any quantity on any infinite set, other than its cardinality in

relation to other infinite sets. Fourth, the endless sequence of natural numbers as an example of a potential infinity suggests, quite nonsensically given Durston's definition, that it is possible to sequentially count toward a process or procedure. Craig's (2008) definition is similarly quixotic:

"*.....a collection that is increasing toward infinity as a limit but never gets there.*"

Although Craig is correctly acknowledging infinity as a collection (or set), he is merely restating here that a growing finite collection or set, no matter how large it may become, will always remain a finite set. So the very notion of a 'potential infinity' is superfluous; it adds nothing to the definition of either a growing finite set or an already existent infinite set. Use of the term 'limit' is also puzzling. By definition an infinite set is boundless and so has no limit. So the idea of something "*increasing toward infinity as a limit*" simply makes no sense. Craig may be misusing a concept from calculus, conflating convergent and divergent functions that tend toward infinity or the infinitesimal with the philosophical notion of a potential infinity. But once again, such functions can never potentially reach infinity because by definition they never do reach infinity; they are asymptotic.

Finally, related to the invalid mathematical concept of potential infinity is the further specious concept of a 'completed infinity' (another term not recognised in set theory, or anywhere else in mathematics for that matter) which Durston (unpublished manuscript) defines as:

"*an infinity that one actually reaches*"

which relies on the twin illogical assertions that, after all, it is entirely possible to count to an infinite set and that some potential infinities actually do reach infinity.

Defining 'Universe'

The concept of the universe is normally used in a highly specific manner by physicists and cosmologists, however, it is commonplace to see words like 'universe', 'world', 'reality' and 'cosmos' used interchangeably by philosophers and theologians. This is an important observation because the validity of any hypothesised first moment in time for the universe (hereafter: *tbeginning*) depends crucially on how the term 'universe' is defined. Two broad definitions are:

1. Everything that we can observe or otherwise known to exist via empirical investigation, i.e., the whole of the causally interrelated material reality within the presently observable spacetime manifold, or:
2. Everything that can exist, i.e., the sum or totality of all existence, in a metaphysically maximal sense. This means that God (and any hypothesised multiverse) is part of the universe, as are any recognised abstract objects, and so it would be illegitimate to refer to anything existing 'outside' or 'outwith' the universe.

Acceptance of either definition does not necessarily delineate theists and non-theists. For example, although most non-theists or philosophical naturalists accept the more basic ontology of (1) some (such as Platonists and Buddhists) will accept the expanded ontology of (2) when deemed to include abstract objects or natural laws or non-material, finitely-existing sentient entities. Most pantheists and panentheists would accept (2). In contrast, classical theists are highly unlikely to accept (2) and so invariably favour (1). This allows them to argue for the necessity of a *tbeginning* and to utilise scientific evidence or hypotheses that might support the idea that the universe began to exist. As the purpose of this essay is to refute a specific claim associated with classical theism, definition (1) is assumed.

Why the Argument Fails Physically

The claim that 'you cannot count back from infinity' relies (especially *via* the Kalām argument), "*from start to finish*" on the A-theory of time (Craig, 1979). The 'tensed' A-theory views time in a similar manner to Plato's 'moving image' with the present moment (hereafter: *tnow*) having been reached via a sequential series of temporal moments (hereafter: *t;* usually referring to a theoretically smallest possible subdivision of time, but not necessarily so). A closely linked concept, 'presentism', is more extreme; it ontologically privileges *tnow* by claiming that whatever can exist can only exist at *tnow*. Although it is possible to be an A-

theorist without necessarily embracing a strict presentism, advocates of the claim that 'you cannot count back from infinity' tend strongly toward a strict presentism. In practice four further characteristics of time are necessary for presentism to work:

1.Each temporal event in the sequence is of identical finite duration
2.The whole of the universe must be included in each temporal event
3.No temporal events overlap
4.There are no gaps between successive temporal events

The claim 'you cannot count back from infinity' therefore commits the claimant to viewing the past timeline, not as a mathematical infinite set, but solely in terms of the philosophical notion of a potential infinity consisting of a linear sequence of *t*s (Craig & Sinclair, 2009). Baldner (1991) agrees:

"*there is.....a sense in which an eternal past might be said to be potentially infinite: if the past were eternal, we could always count more and more past days. We could never count an actually infinite number of them, but we could always (potentially) count more. In this sense.....the past would be, if eternal, a potential infinity.*"

In contrast to the A-theory and presentism, the 'tenseless' B-theory of time ontologically privileges no individual *t* (i.e., there can be no objective *tnow*) and instead maintains that a complete, possibly infinite, array of *t*s exists synchronously. The B-theory enjoys far more support than the A-theory within the physics community because there is ample experimental evidence and mathematically precise models that support the B-theory and very little evidence (if any) to support the A-theory. Most notably, the A-theory is wholly incompatible with special relativity. No evidence exists that there is an objective *tnow* moving sequentially in the same manner at all spatial coordinates within the universe (see e.g., Greene, 2004 for a popular rendition of this research and Petkov, 2009 for a more technical treatment). The A-theory (and presentism) also receive rigorous philosophical critique (see e.g., Mozersky, 2015). Attempts to reconcile presentism with special relativity by invoking an idiosyncratic neo-Lorentzian interpretation of special relativity (e.g., Craig, 2013; Craig & Sinclair, 2009) is an enterprise considered by a majority of physicists and philosophers to be fatally flawed in a number of ways (see e.g., Balashov & Jannsen, 2003; Dorato, 2002; 2003; Mozersky, 2015; Petkov, 2009). In particular, if a neo-Lorentzian universe was the case, the speed of light would be variable. It is ironic, therefore, that Craig argues for a neo-Lorentzian perspective via modern media, such as satellite technology, which depends crucially on the constancy of the speed of light. Not surprisingly, then, Petkov (2009) describes the A-theory as:

"*.....a minority view* [that] *has scraped a meagre existence in the shadows of the major view*."

While Balashov and Janssen (2003) point out that Craig's is:

"*a highly controversial view.....*[equivalent to a] *return to the days before Darwin in biology or the days before Copernicus in astronomy.....Craig fails completely in his attempt to make the case that we should trade in the standard space-time interpretation of special relativity for the neo-Lorentzian interpretation.*"

Similar fare from Dorato (2002) who accuses Craig of being:

"*.....essentially guided by an apologetic attempt and opportunistically uses physics and metaphysics for his purpose.....*[aiming] *to reintroduce in science wild metaphysical hypotheses with no independent support from science.....The evidence for a connection between a neo-Lorentzian interpretation of special relativity and the existence of* [metaphysical] *time.....is very thin.*"

There are two fundamental problems with presentism. The first concerns the relational nature of time itself. On presentism, the perception of *tnow* (i.e., whatever that exists within the universe) continually moves forward in time. So each *tnow* necessarily and continually changes to being a past moment *tnow -n* (where *n* is a natural number). But *tnow -n* is a no-longer existing moment. So, on what basis (other than our subjective perception of *tnow*, and its memory at *tnow +n*) can we assume that *tnow* actually makes such a shift? This question is important because a great deal of the argumentation for the A-theory relies heavily on little more than appeals to human intuition. The influential Christian philosopher Dean Zimmerman (2007) makes no bones about this:

"*My reason for believing the A-theory is utterly banal.....it is simply part of commonsense that the past and future are less*

real than the present.....What it is for some statement to be commonsensical is just for it to seem obviously true to most sane human beings.....Is the A-theory part of commonsense? I think so."

Yet surely there must be some objective measure by which we can make the claim that veridical time actually shifts? There must be some evidence that time moves relative to some other stable dimension or quality wholly unconnected to human psychology. But what is this quality or dimension? It can't be the dimension of time itself that we are experiencing or measuring, obviously. Therefore the presentist needs to posit a further dimension, such as 'meta-time' relative to which time is moving. And this 'meta-time' might then need to be moving relative to some further 'meta-meta-time' and so on, possibly to infinite regress (which, of course, is what the claimant is trying to avoid). In a nutshell, for presentism to be viable at least one privileged, wholly observer-independent objective frame of reference for all of time must exist.

Craig rather simplistically characterises this dichotomy in terms of 'metaphysical time' and 'physical time' and claims, due to their "*positivist bias*", that physicists are merely reading metaphysical implications out of the equations employed to model spacetime. The job of metaphysical time was ascribed to the aether by 19th century physicists but every attempt to identify such a privileged frame of reference has failed, consigned to the rubbish dump of scientific history along with the likes of phlogiston. Theistic presentists have sometimes posited God to be the universal reference medium. But this reduces to a circular argument; positing the necessary existence of God in order to support an idiosyncratic theory of time being employed to support the premises of an argument that is intended to demonstrate the necessary existence of God.

Presentism's necessity for a privileged temporal frame of reference is considerably weakened when analogies are made with the spatial dimensions. Space is naturally isotropic and we have no difficulty viewing space in a relativistic context. Although, similar to time, we do consider ourselves to have a privileged spatial frame of reference, this is done for purely pragmatic purposes. Nobody seriously suggests there is a perfectly veridical spatial coordinate we can label as 'here' which enables us to view other spatial coordinates located at some distance and direction away as somehow less veridical or not existing at all. It is taken as a given that all spatial coordinates exist equally and concurrently, in a similar manner to how the B-theorist views time.

Nevertheless, for the sake of argument, even if we grant the presentist their yet to be identified privileged frame of reference, how is the presentist able to coherently represent change from *tnow*? They cannot consider *tnow* to be a present (existent) *t* and also a (non-existent) future *t* any more than they can state that *tnow* is both a present (existent) *t* and also a past (non-existent) *t*. They could, of course, state that *tnow* is a present (existent) *t* and will be a past (non-existent) *t* at *tnow*+1, *tnow*+2.....However, to maintain the veridical nature of *tnow* notice that they are forced to replace their preferred predicate 'is' with the temporally relative term 'will be'. Here is an example of their problem; the temporally relative term 'will be' as applied to *tnow* cannot be a fact at every *t*. It is only a fact at *t*s prior to and immediately at *tnow*.

In contrast, being temporally invariant, the B-theory of time has no such difficulty with relational concepts because every *t* can be objectively related to any other *t* in a completely tenseless sense. There is no claim of an objective *tnow* existing simultaneously for all observers within the universe. All possible *tnow* differ according to an observer's motion and spatial location relative to a gravitational mass. Observers will not only disagree markedly as to which *t*s are occurring simultaneously they will also disagree as to which physical events currently exist and which do not. Thus, unlike presentism, on the B-theory any possible *tnow* will hold exactly the same temporal relationship to every other *t* throughout the universe. It is perfectly possible, therefore, to consider *tnow* as simultaneously a present moment, a past moment, or a future moment, depending on observers' spatial coordinates within the universe. All of which comports with special relativity.

The second problem for presentism is ontological. As we have seen, because presentism claims that only *tnow* actually exists and that time is strictly tensed, the implication is that whatever can exist can only exist at *tnow*. This is obviously false under a Platonic or neo-Platonic type view of abstract objects such as logic, morality and mathematics. Such things are argued to have no beginning to their existence, will exist eternally, and their continued existence is not contingent on anything other than, perhaps, an eternally existing God (see e.g., Gould & Davis, 2014). To these we may add propositions, properties, and abstract sets or even, according to Plantinga (1976) possible worlds. It is far from parsimonious to contend that such things come into existence or being afresh at each *tnow*. This is no attempt to argue in favour of Platonism. It does serve, however, to illustrate the added burden of proof bedevilling the already beleaguered presentist; there appear to be some things that exist within time yet cannot exist in tensed time.

In addition to an ontologically privileged *tnow* the claimant of 'you cannot count back from infinity' relies heavily on the notion that

time is granular, i.e., it occurs in discrete packets or quanta able to be sequentially enumerated. Physics recognises that some fundamental elements of reality might exist in discrete quanta, but there is little reason to suspect, as yet, that time does so (Hagar, 2014). Indeed, both standard quantum mechanics and special relativity treat time as a continuous variable running from -∞ to +∞ (Carroll, 2008), in a manner similar to the real numbers. If time is indeed continuous, then the temporal duration between any two arbitrarily chosen *t*s would always comprise an infinite set and a shift from *tnow* to any *t* designated as *tnow*+1 would theoretically involve the universe traversing an infinite amount of *t*s in a finite amount of time. Thus, even though the universe may be of a finite age, it does not necessarily follow that there is necessarily a finite number of *tnow-n*, leading to the conclusion that, although there may well be a *tbeginning* there may be no recognisably immediately subsequent *t* (Draper, 2008). This notion is discussed in more detail later.

Craig & Sinclair (2009) do acknowledge that time might be continuous. However, they disagree that the duration between two arbitrarily labelled *t*s comprises an infinite set on the grounds that finite minds can only ever impose finite units onto time. This seems to be an inordinately poor rebuttal, little more than another attempt to privilege human intuition (or more likely, cognitive limitation). Although humans may not be able to directly sense tenseless or continuous time there is no physical, mathematical or logical barrier that prevents us from specifying algorithms able to divide continuous time into any manner of *t*s we wish. Puryear (2014) ironically notes that if we really are only able to divide time subjectively, without recourse to any objective dissection, it follows that time must exist naturally as one continuous, infinite block which is, in essence, the B-theory, and in accordance with set theory. And again, if the past universe comprises one continuous block and this is only able to be divided into discrete events on a solely subjective basis, on what basis can we then aver that a veridical *tbeginning* exists?

Another objection to continuous time is to invoke the hypothesised Planck length (c. 1.65 × 10^35 metres; we are currently able to measure to about 10^20 metres; Hagar, 2014). Before discussing the Planck length it is important to note that the measure is not, as is commonly believed by many non-physicists, the minimum length of spacetime. Indeed, it does not represent the length of anything and, in any case, according to special relativity, lengths can contract. Nevertheless, it is sometimes claimed that calculating the time light takes to travel one Planck length would theoretically identify some minimum *t*. It might be claimed that this minimum *t* cannot be subdivided because it would be impossible to determine the distance between any two spatial locations that are less than one Planck length apart. Four rebuttals can be made. First, the jury remains out with regard to both space and time being granular (Hagar, 2014). There would be profound consequences for physics if it were the case as it would mean that anything measurable within spacetime would also be similarly quantised. In other words, why does this not appear to be the case for all physical phenomena? Oppy (2001) offers this thought:

"*Suppose - for the sake of argument - that temperature is a continuous quantity and that the temperature of an object O increases continually from 150 deg C to 160 deg C over a period of one minute (from t1 to t2). If the temporal series of events is discrete there will be a first moment (M1) after t1. Since the temperature of O increases continuously, the temperature of O at M1 must still be 150 deg C; else there will have been a discontinuous jump in the temperature of O. Repeated application of this argument shows that, at t2, the temperature of O will still be 150 deg C, which contradicts our assumption that the temperature of O at t2 is 160 deg C. In order to avoid contradiction, a defender of the claim that time is discrete must insist that there are no continuous processes.*"

Second, given Oppy's point, is it reasonable to assume that spacetime is granular based only on philosophical musings rather than empirical data? Third, even if we do identify some minimum *t* this might simply reflect our inability to adequately resolve space; the limits of our resolution may or may not represent an objective Planck length. Fourth, even if we did find ourselves capable of resolving to an objective Planck length this may or may not make a difference in terms of mathematics and geometry. It is possible that we could develop mathematical models able to deal with sub-Planck lengths in the same way in which we developed non-Euclidian geometry to deal with measurements on curved surfaces. And if we are able to measure and multiply or divide a single Planck length the Peano axioms should guarantee that we can perform inverse functions on that same operation (as it is with any number that is non-zero). For example, assume four spatial locations each equidistant from each other with any three producing a 90_{deg} angle, representing a square or rectangular configuration. The two diagonals would still measure as a continuous, infinite distance whenever they are represented by an irrational number, whether their units are measured in Planck lengths or not.

Why the Argument Fails Philosophically

We have seen that presentism has difficulties when a relativistic perception of time is required. Not surprisingly, then, presentism finds itself at variance with tense logic (Prior 1967; 1969). The four temporal modal operators are:

P (It will always be the case that it has at some time been the case that.....)
F (It will always be the case that it will at some time be the case that.....)
H (It will always be the case that it has always been the case that.....)
G (It will always be the case that it will always be the case that.....)

If we state the proposition: 'Nero is the Emperor of the Roman Empire', which is clearly false at the moment the reader encounters the proposition, we can nevertheless preserve the truth of that proposition by invoking, for example, the first tensed operator, e.g.,

(Nero is the Emperor of the Roman Empire): is false
P (Nero is the Emperor of the Roman Empire): is true

For this proposition, the P operator works equally as well under presentism as the B-theory. Now, if we state the proposition: 'there are *tnow-n*', on the B-theory this statement would be true and of course it remains true if we invoke the tensed operator P:

(there are *tnow-n*): is true
P (there are *tnow-n*): is true

However, under presentism:

(there are *tnow-n*): is false
P (there are *tnow-n*): is false

and invoking the modal operators F, H or G would be of no help to the presentist. While it is a matter of historical record that Nero was the Emperor of the Roman Empire at some *t*s, *tnow-n* simply cannot exist for the presentist. Now consider the proposition: 'there will be *tnow+n*. On the B-theory:

(there will be *tnow+n*): is true
P (there will be *tnow+n*): is true

However on presentism:

(there will be *tnow+n*): is undetermined
P (there will be *tnow+n*): is false

Even if we were to grant the presentist that all *tnow-n* are non-existent, for the purposes of making the claim 'you cannot count back from infinity' they must commit themselves to the notion that what is non-existent is nevertheless able to be enumerated. Thus, an ordinal relationship should be discernible between *tnow* and all *tnow-n*, such as *tnow -1, tnow -2*.....or, coming at it from the other direction, *tbeginning* +1, *tbeginning* +2.....Their principal deduction is that if an infinite temporal regression was the case it would be impossible for the universe to ever reach *tnow*. As Craig (2008) asserts:

"*If the universe never began to exist, then prior to the present event there have existed an actually infinite number of previous events. Thus, a beginningless series of events in time entails an actually infinite number of things, namely, events.*"

And from Craig & Sinclair (2009):

".....before the present event could occur, the event immediately prior to it would have to occur; and before that event could occur, the event immediately prior to it would have to occur; and so on ad infinitum. One gets driven back and back into the infinite past, making it impossible for any event to occur. Thus, if the series of past events were beginningless, the present event could not have occurred, which is absurd."

Mathematically, this kind of reasoning should raise a red flag straightaway. Any assertion that *t*s can be sequentially counted using

ordinals immediately presupposes that an infinite past does not obtain. If *t*s are sequentially countable there could only ever be a finite number of them because, as mentioned, at no point in an infinite regress will any finite number have an infinite number as its successor. This approach therefore commits the logical fallacies of 'circular argument' and 'begging the question'. In effect, *tbeginning* is being postulated to exist *a priori* in order to counter the possibility that there is no *tbeginning*. Thus, these contradictory premises:

(i) An infinite amount of *t*s comprise an infinite set of *t*s
(ii) This infinite set of *t*s must be bounded by *tbeginning*, a first cause event in the past which can also be labelled *tnow -n* (or worse, *tnow* - ∞). This presupposed *tbeginning*, along with *tnow*, explicitly bounds and quantifies an infinite set (i.e., from *tbeginning* to *tnow*), thus rendering it finite.

If (i) is true (and mathematically it is), then (ii) is contradictory, simply because an infinite set is defined as being unbounded. This contradiction (or absurdity, to use Craig's favourite term) is then presented as a sound conclusion that an infinite past is impossible on the grounds that it leads to such contradiction and absurdity. But this is obviously wrong-headed. That you cannot successively add or subtract your way to an infinite set is part of the very definition of an infinite set, temporal or otherwise. Craig and Sinclair (2009) actually acknowledge this in another version of their argument:

P1: A collection formed by successive addition cannot be an actual infinite
P2: The temporal series of events is a collection formed by successive addition
C: Therefore: the temporal series of events cannot be an actual infinite

P1 is true and so acknowledges that even the attempt to enumerate past *t*s renders the past finite. Someone who accepted that the past comprised an infinite temporal set would not even entertain the notion that past *t*s could be sequentially numbered. P2 consolidates the notion that past *t*s comprise a finite set. This argument rests, of course, on time being granular. But it is interesting also to note how much this kind of argument has evolved, in response no doubt to numerous rebuttals, evidenced by a progression from "*equal past intervals of time*" to "*number of things*" to "*a collection formed by successive addition*". In 2001 Craig had constructed the argument in this way:

P1: An actual infinite cannot exist
P2: A beginningless series of equal past intervals of time is an actual infinite
C: Therefore: a beginningless series of equal past intervals of time cannot exist.

Later in 2008, he offers:

P1: An actually infinite number of things cannot exist
P2: A beginningless series of events in time entails an actually infinite number of things
C: Therefore: a beginningless series of events in time cannot exist

The shift from "*past intervals of time*" to "*number of things*" changes the very definition of what Craig is arguing should be countable. Arguably, temporal moments, events or intervals of time do not exist in the same way as 'things'. We can successively count many things using a direct representation between number and thing but we cannot sequentially count temporal moments except as they correspond to some other thing, i.e., some type of clock. In the absence of any measuring device, how many temporal events or moments can we claim have occurred in the past chunk of time, for example? The 2001 and 2008 versions of Craig's argument are therefore inconsistent. On the one hand, Craig (2001) differentiates between 'events' and 'things' when he states:

"*.....it is things, not events, that come to be; an event is just the coming to be of some thing or things*"

yet in P2 of the 2008 version of his argument he infers that because a temporal event "*entails*" a 'thing' the two can be equated. But how does a *t* necessarily entail a 'thing'? Further confusing the issue, the latest version of the argument (Craig & Sinclair, 2009) has the concept of 'entailing' dropped and a temporal event redefined not as a thing but as "*any change*". However, as Hedrick (2014) has argued, change is something that happens to a 'thing', through losing, gaining or altering its properties, whereas *t*s, whether conceptualised as "*equal past intervals of time*" or 'temporal events', are such fundamental aspects of reality that they have no such properties to gain or lose. Furthermore, a number of philosophers have denied that temporal events even exist at all

(e.g., Horgan, 1978; van Inwagen, 2011). Craig (2011) partly acknowledges this in his description of temporal events as:

"*.....the sorts of thing that many metaphysicians plausibly deny exist..... these things are real in the sense that they are not illusory, but they are not, properly speaking, existents.*"

Thus Craig turns down the convoluted *ad-hoc* avenue favoured by the theologian. Although temporal events are "*not illusory*" they are not "*existents*". However, they must be logically possible, evidenced by the attempt at enumeration, yet they are somehow metaphysically impossible. As Dorato (2002) observes, Craig sometimes exhibits an:

"*uncontrolled taste for metaphysical speculations."*

Indeed, when evidence from physics clashes with his assertions regarding infinities Craig often deflects the argument toward metaphysics. From a debate with philosopher Peter Millican (Millican & Craig, 2011):

"*.....the real existence of an actual infinite number of things leads to metaphysical absurdities.*"

And similarly (from Craig, 2001):

"*relativity physics.....is not necessarily saying anything that is relevant for the metaphysician*"

The fact that Craig even feels the need to devolve arguments regarding physics into metaphysical speculation shows that he has been forced onto the back foot. As we shall see the "*real existence*" of infinity (whatever 'real existence' is as opposed to just 'existence' is not made clear) does not necessarily imply physical or logical absurdities, these arise only when we attempt to manipulate infinities in illegitimate ways (Morriston, 2013; Woodin, 2011a, 2011b; Hauser & Woodin, 2014). Craig seems to be implying that although metaphysical possibilities must always be logically possible, the reverse is not true and logically possible things (such as a past eternal universe) are not necessarily metaphysically possible. This view lacks coherence. We are able to distinguish between logical and physically lawful possibilities well enough (even though classical logic, at least, might be scale-variant, e.g., Bueno & Colyvan, 2004) but how do we even begin to separate logic from metaphysics? On what possible grounds could someone acknowledge that a past eternal universe is physically and logically possible yet cannot obtain because it is metaphysically impossible? Relying on personal incredulity as a gauge to metaphysical assertions and then employing those metaphysical assertions to uphold further metaphysical assertions while ignoring contrary logical and physical evidence because it offends our intuitions is another circular methodology and a thinly veiled attempt to concoct not only an unfalsifiable system of knowledge but one in which the theologian alone holds sway and the physicist is *persona non grata*.

Why the Argument Fails Mathematically

As discussed, despite temporal events being "*not existents*" to support the claim 'you cannot count back from infinity', they need to be at least theoretically countable. It is obvious, therefore, that although Craig and Durston are both proponents of the claim, Craig and Sinclair's 2009 version of P1:

"*.....a collection formed by successive addition cannot be an actual infinite.*"

is at odds with Durston's statement (unpublished manuscript) that an infinite temporal regression would mean that:

"*.....the number of seconds in the past is a completed countable infinity.*"

Here we see Durston making the common mistake of equating an infinite set with some very large number. Craig and Sinclair are, of course, correct in this instance. An infinite set is never defined as such solely by how many elements it contains. Durston (unpublished manuscript) appears to be confused about the whole matter of infinities. After first defining a 'completed infinity' as "*an infinity that one actually reaches*" and telling us that "*a completed infinity is a set*" he then contradicts himself by stating that it is "*impossible to count to a completed infinity*" and "*the number of elapsed seconds in the future is a potential infinity*". At this point we shall leave Durston to his obvious confusion.

But even if the A-theory was true, mathematically speaking it would remain the case that an infinite temporal set is not achieved by successive addition. This is because, as we have seen, there is no legitimate mathematical operation that allows counting to (or from) an infinite set by successive addition or subtraction. 'Counting to infinity' is in no way analogous to counting to a trillion or a googolplex. Similarly, 'counting back (or forward) from infinity' is in no way analogous to counting events forward from *tbeginning* or backward from *tnow*. The mistake being made (and as we shall see, is made all too often) is a failure to understand that an infinite set never corresponds to any number. Infinity is not a number. It is a property held by numbers and you cannot perform standard arithmetic inverse operations such as such as 'counting forward' or 'counting back' on the properties held by numbers. To illustrate:

Let {N} be the infinite set of all natural numbers. If we attempt to subtract all the even natural numbers from {N} we effectively have:

$\infty - \infty = \infty$

because the odd numbers that remain also constitute an infinite set. Similarly, we would also have:

$\infty + 1 = \infty$
$\infty * 10 = \infty$
$\infty * 10^7 = \infty$
$\infty * 10^7 * 5 = \infty$
$\infty + \infty = \infty$
$\infty * \infty = \infty$

And so on. It matters not whether the members of an infinite set are natural numbers, existent or non-existent moments in time or worldly things such as grains of sand. These are not contradictory results. They are the only possible answers under these dubious mathematical circumstances. Standard inverse arithmetic operations such as addition, subtraction, multiplication and division as formulated in the Peano axioms underlie number theory. But they are not defined for use with infinite sets. The results in the arithmetic operations above only appear absurd if you expect to be able to perform standard arithmetic operations on infinite sets with the expectation that they should produce similar results to adding and multiplying finite integers. $\infty + 1 = \infty$, for example, is only absurd if you hold to the mistaken believe that $\infty + 1$ is somehow a larger amount than ∞ alone. Despite this fact, Craig & Sinclair (2009) argue that if the past constitutes a temporal infinity:

".....then there have occurred as many odd-numbered events as events. If we mentally take away all the odd-numbered events, there are still an infinite number of events left over; but if we take away all of the events greater than three, there are only four events left, even though in both cases we took away the same number of events."

In other words, according to Craig and Sinclair, in this particular case:

$\infty - \infty = 4$

Craig and Sinclair are no doubt relying on their audience being naïve to set theory and defaulting to their intuitions or, more accurately, their sense of incredulity. But mathematics does not rest on intuition. It rests on a formal logical system. Once again, there is nothing intrinsically problematic about infinite sets; it is the unlawful ways we might manipulate these sets that creates apparent absurdities (Hauser & Woodin, 2014; Morriston, 2013; Woodin, 2011a; 2011b). Attempting to simply subtract the infinite set of numbers > 3 from {N} is a gross mathematical error. Craig and Sinclair are misusing the concept of infinity by treating both an infinite set and its infinite subset as if they were numerical quantities representing a finite number of elements. But infinity is a property of numbers and you cannot manipulate infinities in the same way you can manipulate finite numbers or finite sets of objects. It is not simply the case that infinities are, for pragmatic reasons, treated as a specific exception to number theory; infinities are in no way part of number theory. If we allow ourselves to apply the Peano axioms willy-nilly to infinite sets we produce absurdities such as this:

If $\infty + 1 = \infty$
and

$\infty + 2 = \infty$
we can reasonably deduce that:
$\infty + 1 = \infty + 2$
now if remove the ∞ from both sides we get
$1 = 2$
and if we subtract 1 from each side we get
$0 = 1$

Although this kind of arithmetical procedure is perfectly coherent with finite numbers, it is obviously illegitimate when dealing with infinite sets. But it being so does not mean that an infinite temporal regression is impossible. It means that the arithmetical operations allowed by the Peano axioms are only legitimate under specific, well-defined mathematical conditions. There is nothing mathematically awry about infinite regressions and there exist no mathematical proofs discounting infinite sets (Woodin, 2011a). This does not mean that an infinite set cannot be split, however. Let:

{A} = the infinite set of all natural numbers;
{B} = the infinite set of all even natural numbers;
{C} = the infinite set of all natural numbers >3;
{D} = the infinite set of all odd natural numbers;
{E} = the finite set of natural numbers < = 3.

Because each element in {B}, {C} and {D} can be placed in one-to-one correspondence with an element of {A} it follows that {B}, {C} and {D} are each a subset of {A}. Also, because each element in {*B*} is in one-to one-correspondence with each element in {*C*} then, in terms of cardinality, {*B*} = {*C*}. We can now perform the following operations:

{A}\{B} = {D}
Which means: if infinite set {B} is removed (note: not subtracted) from infinite set {A} then infinite set {D} will remain, which is what Craig and Sinclair (2009) have done and claimed to have produced an absurd result, seemingly disregarding the fact that {A} = {D} because {A} and {D} have the same cardinality.

However, despite {*B*} = {*C*}:
{A}\{C} = {E}
Therefore: {*A**B*} ≠ {*A**C*}

This result is not absurd for the following reason: we cannot subtract an indeterminate number of elements (such as Craig's set of natural numbers > 3) from an infinite set that is also of indeterminate number (such as all of the natural numbers) but we can remove a finite subset of elements from an infinite set to create a second, finite set if we specify in advance precisely which of the finite number of elements from the infinite set are going to be removed (East, 2013; Hinman, 2005). In this case by removing {C} from {A} rather than {B} from {A} we are left with two sets, the infinite set {C} whose cardinality is identical to {A} and the removed remainder of {A} which is now the finite set {E} or {0, 1, 2, 3}. There is no mathematical contradiction here because the arithmetical operation of subtraction has not occurred. Consider, then, the following three statements:

S1: It is not possible to precisely define $\infty - \infty$; but
S2: We should expect to be able to perform the arithmetic operation of subtracting an indeterminate number of elements from an infinite set to achieve a finite set.
S3: We can remove a finite number of elements from an infinite set to achieve a second, finite set if the exact elements being removed are specified in advance.

S1 is true. Any answer would be indeterminate. S2 is plainly false; yet this is what those who claim 'you cannot count back from infinity' expect should be the case when they attempt to count back to infinity only to find it produces absurdity. If S2 was not false they should just as legitimately be claiming that an infinite temporal regress is impossible because 'you can't count back from the infinite set of even numbers' or 'you can't count back from the infinite set of prime numbers'. S3, of course, is true as we have shown with {A\C} = {E}.

As alluded to, whenever the alleged absurdities regarding infinities are employed in service of some theological goal some appeal to

intuition is invariably made. For example, Craig (2008) supports his premise that 'the temporal series of events is a collection formed by successive addition' by stating that this is "*obvious*". This rests on shaky ground for three obvious reasons. First, when has human intuition ever been a reliable guide to truth? Science alone has surely taught us otherwise. Any number of examples can be marshalled in evidence, such as the strong perception that the ground under our feet is stationary or measurable changes in the mass of an object relative to both its motion and the relative coordinates of the observer. Second, our intuitions regarding mathematics have evolved through experience with finite numbers representing finite sets of objects. We are so committed to quantising finite objects in terms of finite perimeter conditions such as 'best', first', greatest' and 'maximum' that conceptual spaces beyond 'best' and first' do not fit comfortably, even within mathematics and logic (Oppy, 2006). Moreover, our intuitive understanding of mathematics is often inconsistent and contradictory. For example, it is rare to find someone with no mathematical training who is able to accurately define the concept of 'finite'. Yet some people with no mathematical training can give a reasonable definition of infinity (Suber, 1998). The fact that we cannot enumerate or even conceive of all the distinct elements within an infinite set is an irrelevant truth here; we cannot conceive of all the distinct elements within the vast majority of finite sets either. A chiliogon (a 1,000-sided regular polygon), for example, is a perfectly computable and physically realisable geometric shape but it is impossible to visualise. Third, there is more than a hint of sophistry here. Applying intuitions about finite numbers to infinities because finite numbers accord with our empirical observations disregards the formal logical bases by which infinities become meaningful concepts. Despite the availability of consistent mathematical-logical mechanisms, human intuition regarding infinities is being represented as the more legitimate *a-priori* metaphysical principle. Cantor (1955) recognised the fallacious nature of this view in his letter to the mathematician and historian Gustav Enestrom:

"*All so-called proofs of the impossibility of actually infinite numbers are false in that they begin by attributing to the numbers in question all the properties of finite numbers, whereas the infinite numbers, if they are to be thinkable in any form, must constitute quite a new kind of number as opposed to the finite numbers, and the nature of this new kind of number is dependent on the nature of things and is an object of investigation, but not of our arbitrariness or prejudice.*"

The difficulties many people encounter when dealing with infinite sets likely reflects the limitations of our cognitive mechanisms. This is especially evident when Cantor's Principle of Correspondence meets Euclid's Maxim. Craig appears to be capitalising on mismatches such as this. This is exemplified in a story told by Craig (1979; though quoted verbatim here from Craig, 2015):

"*So, Ghazali says, let's imagine our solar system, and here is Saturn. And let's imagine that for every one orbit that Saturn completes, around the sun, Jupiter, which is closer in, completes two.....Now, notice that the longer they orbit, the further Saturn falls behind. If Jupiter has done ten-trillion orbits, Saturn has only done five-trillion, and the longer they orbit, the farther and farther Saturn falls behind. If they continue to orbit forever, they will approach a limit at which Saturn is infinitely far behind Jupiter.....Now, let's turn the story around, says Al Ghazali. Suppose that they have been orbiting the sun, from eternity past, now which one has completed the most orbits? Well, the answer mathematically is that the number of orbits completed is exactly the same.....As I say, this is his argument, in the 12th Century. It's just amazing to read this stuff.....You can't get out of this argument by saying that infinity isn't a number, because it is a number in this case. We're dealing with an actually infinite number of orbits."*

Although a similar paradox did originate with the medieval Islamic philosopher al-Ghazali, Craig is incorrectly attributing this particular version to him, apparently unaware that heliocentrism was unknown at the time of al-Ghazali's writing in the 12th century. It's not the only thing he got wrong. There is simply no mathematical rationale for claiming that infinity is "*a number in this case*." Infinity is never a number. It is a property of numbers. Nevertheless, Craig goes on:

"*Al Ghazali asks, "Is the number of orbits completed odd or even?" And, you know, what the answer is, mathematically– it's both. It is! It's both odd and even. So, that again just shows, I think, the absurdity of trying to form an actually infinite number of things by successive addition.*"

This assertion is misleading. A typical definition of an even number is $n = 2k$ where k is an integer. The definition of an odd number is therefore $n = 2k + 1$. It is only integers themselves that can possess the property of being even or odd. Because an infinite set is not a number, never mind an integer, it cannot be considered even or odd. Furthermore, all numbers that are not integers can be neither even nor odd and not only is there an infinite set of them but the cardinality of that infinite set is higher than the cardinality of the infinite set of integers. There is no paradox here at all once it is understood that the infinite orbital counts of the two planets can be mathematically distinguished, not by ordinality but by their differing cardinality (Oppy, 2006). Again, Craig's claim of

absurdity rests on his viewing infinity as a number and considering the defining property for an infinite set to be the number of elements it contains. He is attempting to redefine infinity in terms of finitude. In finite arithmetic we can identify even numbers by starting with any set of natural numbers and subtracting every number that satisfies $n = 2k + 1$. If we do this, the remaining even numbers will obviously be less numerous than our set of natural numbers. But this does not work with infinite sets because an infinite set is properly defined as a set of elements where the elements of a proper subset (in this case, odd or even numbers) can be put into one-to-one correspondence with the elements of the whole set. If the entirety of Jupiter's and Saturn's orbits were a finite set then, yes, a contradiction would result. But they are not a finite set. To complain that the infinite subset of Jupiter's orbits is the same size as the infinite subset of Saturn's orbits you must deny the very definition of an infinite set. Unsurprisingly, Craig (2010a) is not convinced by the logic of set theory:

"*These developments in modern mathematics merely show that if you adopt certain axioms and rules, then you can talk about actually infinite collections in a consistent way, without contradicting yourself.*"

"*Merely*" is the word doing all the heavy lifting for Craig's derision. Remove it and the statement's meaning changes completely. One wonders, then whether Craig would be so keen to include the same word in this slight rewording of his quote:

"*These developments in modern theology* [merely] *show that if you adopt certain presuppositions and logical rules, then you can talk about an actually existing God in a consistent way, without contradicting yourself.*"

And what of these mathematical rules for dealing with infinities? There are two choices. The first, set theory, is logically consistent and non-contradictory and the second, finite arithmetical procedures, returns logically inconsistent and contradictory results and, by Craig's own reckoning, absurdities. So which of these two does Craig choose to employ when employing infinities to make his case? The latter. But claiming that a temporal infinite regression cannot be the case because standard arithmetic procedures do not work on infinities is surely no more plausible than stating that a zero quantity of something is impossible because:

0 - 0 = 0
0 * 0 = 0
0/0 = undefined/uncomputable
1/0 = undefined/uncomputable
0/2 = undefined/uncomputable

and equations with a missing numeral, such as:

0 *_ = 0

have no unique solution; indeed they all have an infinite number of possible solutions. But then, as we have seen, so does this equation:

∞ * _ = ∞

Despite the apparent absurdities resulting from procedures involving zero surely no-one would seriously claim that zero does not exist as a coherent mathematical entity. Quite the opposite. There is much nonsense surrounding zero in the popular literature such as Seife's (2000) hyperbolic contention that:

"*.....within zero there is the power to shatter the framework of logic.*"

If the universe does constitute an infinite temporal set then according to the B-theory of time (and set theory) it must have always constituted an infinite set. It could neither have spatially expanded nor aged itself to infinity. It would have come into existence as an infinite set, in which case there would be no individual *t* or temporal relations between multiple *t*s that would demonstrate otherwise. As discussed earlier, within any temporal continuum (i.e., a continuous range of points between, but not including, any two *t*s) there must also exist an infinite temporal set. Imagine then that an angel has been 'counting forward from infinity' from the depths of eternity and is now finishing the sequence (for whatever reason). Let's say she has been counting at a rate of one natural number per second for infinite time. She now decides to stop counting. It makes no difference whether this terminus is labelled *tnow* or *tnow-1* or *tnow- 1trillion* or whenever. Regardless of when she finishes an infinite amount of seconds will still

have elapsed. It makes no difference whatsoever that *tnow* is a different number to *tnow-1* because all *t*s hold the property of being equally valid elements within an infinite temporal set. Although our angel would have been (somehow) 'counting forward from' (or more correctly for) an infinite number of *t*s she would (at whatever *t*) always have infinitely many numbers left to count had she not decided to end the count. And whenever she ended the count she could justifiably label *tnow* because she would only ever have been counting *within* an infinite temporal set and never *toward* an infinite temporal set. The fact that a temporal regression is infinite does not preclude any *tnow* from occurring or being experienced if the regression extends both backward in time infinitely and continues to progress infinitely into the future. If, at any *t*, she provided a progress report, our angel could accurately respond that she had counted infinite past *t*s and had infinite future *t*s left to count.

Thus, claiming that it is implausible that we have arrived at *tnow* by traversing an infinite temporal regression (and so there must be a *tbeginning*) does not negate the possibility that the time elapsed since *tbeginning* comprises an infinite temporal set. Imagine a straight horizontal line in space that is infinite in length and divided up into millimetre marks (spatial moments, if you like). It would be absurd to sequentially shift one's attention along the line for eternity, stop at some random point, analogously label it as *spatialnow* and then claim that it is impossible to have reached this mark. It is absurd because wherever you stopped would be *spatialnow* and there would always be infinitely many millimetre marks to the right and left. Returning to the temporal domain, Draper (2008) remarks:

"*If the temporal regress of events is infinite, then the universe has never had a finite number of past events. Rather, it has always been the case that the collection of past events is infinite. Thus, if the temporal regress of events is infinite, then the temporal series of events is not an infinite collection formed by successively adding to a finite collection. Rather, it is a collection formed by successively adding to an infinite collection. And surely it is not impossible to form an infinite collection by successively adding to an already infinite collection*."

Countering evidence such as this requires a strict finitist view which negates the very notion of the existence of infinite sets of any kind:

P1. An actual infinity cannot exist
P2: An infinite temporal regression would be an actual infinity
C: Therefore: An infinite temporal regression cannot exist

Importantly, arguments such as this are not based on empirical data. As Craig (2011) states:

"*The primary argument[s] that I give for the finitude of the past are philosophical arguments, based on the impossibility of the existence of an actually infinite number of things, and then secondly on the impossibility of forming an actual infinite by successive addition. So I see the scientific evidence as merely confirmatory of a conclusion that has already been reached on the basis of philosophical arguments*."

The scientific evidence Craig refers to is, of course, the A-theory of time along with his neo-Lorentzian interpretation of special relativity. Nevertheless, in the absence of empirical evidence, several thought experiments have been fashioned to illustrate the alleged absurdities that might occur if an infinite set existed. The paradox most favoured by 'you cannot count back to infinity' claimants is that of Hilbert's Paradox of the Grand Hotel (Gamow, 1947). There are a number of slightly different formulations of this paradox but all convey the same idea. Hilbert's Hotel has an infinite amount of rooms, consecutively numbered, all of which accommodate a single person and so an infinite number of guests are present. Someone arrives at the hotel looking for a room and in order to accommodate him the manager shifts everyone into the next room. So the guest in Room 1 moves to Room 2 and the guest in Room 2 moves to Room 3, *ad infinitum*. The new guest is then able to be accommodated in room 1. In spite of this additional guest the amount of guests in the hotel remains the same. In this case, once again, it appears that:

$\infty + 1 = \infty$

Suppose that a guest decides to leave and her room becomes vacant. The amount of guests in the hotel is unchanged, because:

$\infty - 1 = \infty$

Then, if all the guests in the odd numbered rooms decide to check out the hotel would still have an infinite amount of guests even

though an infinite number of guests would have left, because:

$\infty - \infty = \infty$

However, according to Craig this situation is absurd because if all guests in the rooms numbered 6 and above were to check out, then exactly five guests would remain. So in this case:

$\infty - \infty = 5$,

a tactic, as we have seen, that Craig regularly employs to demonstrate that infinities cannot exist. From Craig (2008):

"*It is indisputable that if an actually infinite number of things were to exist, then we should find ourselves landed in an Alice-in-Wonderland world populated with oddities like Hilbert's Hotel.*"

But it is not indisputable. All this is merely a rehash, involving the same mathematical errors explained earlier. It is perfectly acceptable to remove a finite number of guests from the infinite set that is Hilbert's Hotel (e.g., all guests in rooms numbered 5 and below) because we would be specifying exactly which guests were checking out. It is not acceptable, however, to subtract the undetermined number of guests in the rooms numbered 6 and above from the infinite set that is Hilbert's Hotel because the actual number of guests checking out cannot be specified (infinity is not a number!) and so arithmetic operations such as subtraction cannot be performed. Again, Craig misrepresents the mathematical operations that are allowable with infinities. He tries to convince us that it is paradoxical that we can have two infinite sets with equal cardinalities and differing yet indeterminate numbers of elements (e.g., every room occupied vs. only the even numbered rooms occupied). He's not alone in his confusion. This sentence comes from Wince (2009):

"*If you use Cantor's set theory to explore numbers, you get to the uncomfortable result that there are different sizes of infinity.*"

But on what grounds would someone who understood mathematics "*use Cantor's set theory to explore numbers*" in the first place? And whether or not we get an "*uncomfortable result*" is wholly irrelevant to the soundness of the formal-logical system of set theory. To labour these points to destruction; infinity is not a number, it is a property held by numbers and infinite sets do not come in a 'one size fits all' package. They differ in size. Looking at it in terms of primality might be helpful. Some numbers have the property of primality but primality itself is not a number. It is a property held by some numbers and the amount of numbers exhibiting this property is infinite. Imagine, then, Hilbert's Prime Hotel containing a prime number of rooms in which a prime number of guests checks out. Assuming each room checking out was only ever one of a twin prime (see e.g., Zhang, 2014) in every case the hotel will be left with a prime number of guests. Yet in every case the actual number of guests will not differ before and after check-out. Nothing absurd here.

Ironically, al-Ghazali (unwittingly) semi-dealt with Craig's objections to Hilbert's Hotel nearly a millennium ago when he postulated (see McAllister, 2011) that if God created an immortal angel every day and if the universe they lived in had existed for an infinite number of days then the result would be an infinite, co-existing population of such immortal beings. This, of course, is not really true as a growing series can never reach infinity; al-Ghazali's knowledge of infinity derived from Aristotle. Nevertheless, al-Ghazali did realise that if God had created an infinite set of immortal angels they would be able to arrange themselves into one infinitely long line. Then, on his command, they could rearrange themselves into two or three lines each of infinitely long yet differing length. Or, he could have an infinite amount of angels congregate randomly so only a specific number of angels remained neatly in line, thereby removing a finite set from an infinite set. Al-Ghazali's domain of an infinite population of immortal angels being shuffled around is analogous to a celestial Hilbert's Hotel.

There are further problems for those denying outright the possibility of an infinite set. They would first need to specify those properties of an infinite set that would always be unobservable under any circumstances. They would then need to identify the discoverable, observable boundaries that would automatically disqualify any set from being an infinite set. As discussed, simply identifying one boundary condition, such as the natural number 1 or *tbeginning* does not necessarily preclude an infinite set. As a thought experiment Hilbert's Hotel meets none of these requirements.

It has been argued that the real strength of Hilbert's Hotel lies in its physicality, i.e., because it demonstrates that this kind of

infinite set is physically impossible then the notion of an infinite temporal universe is defeated. However, a specific example of a physical impossible infinite set does not mean that all physical infinite sets are equally impossible. Hilbert's Hotel is physically impossible, granted, but Oerter (2010) has formulated an astronomical scenario analogous to Hilbert's Hotel which although physically highly improbable, is certainly not either physically or logically impossible. He starts by substituting galaxies for guests and employs ordinal numbers as identifiers of these galaxies instead of room numbers. He then asks us to imagine a spatially infinite universe containing an infinite number of galaxies (i.e., an infinite set whose elements are galaxies). From a specific location on Earth a beam, which travels endlessly, is emitted in a straight line into space. Naturally, this beam can intersect an infinite number of galaxies and at any one time we can number those galaxies which are intersected, e.g., the first galaxy the beam intersects is 'Galaxy 1', and the next is 'Galaxy 2' etc. *ad infinitum*.

The universe, of course, is not static, so galaxies and Earth are continually in motion. If an infinite number of galaxies are intersected by the beam at *tnow* it is physically possible that at *tnow+1* a new galaxy has shifted position and become intersected by the beam. It is also physically possible that the new galaxy is closer to Earth than 'Galaxy 1' was at *tnow*. Thus at *tnow+1* we would need to renumber all the intersected galaxies. The new galaxy becomes 'Galaxy 1'; the previous 'Galaxy 1' is relabelled 'Galaxy 2' etc. *ad infinitum*. How many galaxies are intersected by the beam at *tnow+1*? An infinite set. Now imagine that, at *tnow+2*, all the even numbered galaxies are no longer intersected by the beam, but all the odd-numbered galaxies remain intersected. How many galaxies are intersected by the beam at *tnow+2*? An infinite set. The arrangement of galaxies along the beam at *tnow+1* is directly analogous to an additional guest arriving at Hilbert's Hotel and the arrangement of galaxies at *tnow+2* is directly analogous to all the guests in even numbered rooms checking out. Craig's argument is that because Hilbert's Hotel is physically impossible it is also metaphysically impossible. Oerter's (2010) analogy, on the other hand, is physically improbable, but it is not physically impossible; so on what grounds is it metaphysically impossible?

Simply treating infinity as a theoretically countable quantity and offering examples of logical contradictions in highly specific, carefully selected cases such as Hilbert's Hotel does not provide sufficient evidence that infinite sets are forever impossible in all domains. Hilbert's Hotel is comprised of physical objects such as rooms and guests yet there is not even a requirement that infinities be physical. Recall that Craig (2001) sometimes finds the need to differentiate between 'things' and 'events' when arguing against infinities, as did Cantor (1887) who argued for infinities both *in concreto* and *in abstracto*. Thus, if a physical object such as a universe is finite it does not preclude that there are elements within that universe, such as time, that are continuous and infinite. Furthermore, if abstract objects such as numbers are considered as things then quite obviously Hilbert's Hotel fails to show that infinities are impossible or absurd. And the irony may be that this best candidate for an infinite set may necessarily be immune to Craig's alleged absurdities. Hedrick (2014) proposes (somewhat tongue in cheek) 'Hilbert's Platonic Heaven' in which all numbers reside in their perfect forms. He then goes on:

"*Now suppose a new number comes along wanting to be added to the collection. In this case, we simply need to shift all of the other numbers around in such-and-such a way, and as a result we will have accommodated the new number. Or suppose a number decides to leave this Platonic Heaven. Nevertheless, the same number of numbers remains as before.*"

Mathematics, however, does not concern itself with metaphysical speculation, nor with physical or abstract objects *per se* but with the possible relationships between those objects. Abstract objects cannot be moved around, either naturally or intentionally, in the same way as physical objects. So, if the hotel's guests stayed in their rooms and didn't bother to check out, no absurdities would arise and the infinite set of Hilbert's Hotel can exist in metaphysical comfort. Something else that cannot check out or be shuffled around are *t*s. This makes Craig's use of Hilbert's Hotel disingenuous in another important way. Previously, the argument against an infinite series of temporal events or past moments was made on the basis that it would be impossible to reach *tnow* by sequential count. Astute readers, then, will have noticed that Hilbert's Hotel does not model an infinite temporal regression and so represents a poor analogy. Hilbert's Hotel is a theoretical example of an infinite set, existing at *tnow* in which all the elements (rooms, guests) must exist concurrently. However, under presentism the *t*s making up a temporal sequence can never exist concurrently, only successively. Even if it were possible to subtract from *tnow* past *t*s (that according to presentism do not exist) it is not at all possible (and would be a pointless exercise) to shift *t*s around like the guests in Hilbert's Hotel in order to make the point that an infinite set of them cannot exist. Craig, of course, disagrees (cited in Morriston, 2012):

"*You can still imagine what it would be like for a person in room one to be in room two, the person in room two . . . can be in room four, and you can generate the same absurdities. You don't have to go to the trouble of moving the persons physically.*"

Once again, Craig is grasping at (metaphysical) straws. Yes, even if it is physically impossible for the guests of Hilbert's Hotel to

shift rooms we can still use our imagination as to what would result if they actually did shift rooms but I'm not sure what point is being made. Plenty of things are possible in our imagination without holding any physical or logical consequence. Escher's drawings can be perfectly well imagined, for example. We can't go to the trouble of actually shifting *t*'s around either, so does anything of consequence really entail if we imagine that we have done so? What the Hilbert's Hotel paradox really demonstrates is not that an infinite set cannot exist, but that we cannot use the same arithmetical procedures on infinite sets that we can apply to finite numbers and sets.

A Final Nail in the Coffin: Infinite Progression

If the universe is granular and, with the exception of *tnow* comprised of discrete *t*s which, although non-existent, can nevertheless be numbered, then the discrete *t*s that make up the future are likewise non-existent yet enumerable. Future *t*s are "*equal intervals of time*" to past *t*s (it would certainly take some novel evidence and ingenious argumentation to claim otherwise) so there would be no reason why we cannot enumerate an endless progression of future *t*s in the same manner as past *t*s and attempt to perform the same arithmetical operations on them as with an endless regression of past *t*s. If we do this and similarly generate contradictions and absurdities it surely follows that an infinite series of *t*s, whether regressing into the past or progressing into the future, must be equally absurd. Consider Morriston's (2012) 'Angelic Praise Scenario':

"*Suppose that God has just decreed that Gabriel and Uriel will take turns praising Him for one minute of celestial time, and that they will do so forever. Gabriel will do the odd-numbered praises and Uriel the even-numbered ones. Let's go a step further. So as not to leave any opportunity for Gabriel and Uriel to mess things up, let's suppose that this is no mere instruction or recommendation, but that God has exercised His supreme power in such a way as to make it the case that each praise in the endless series of praises we have envisaged will occur. Each of them is discrete, wholly determinate, and certain to occur because God has determined that it will occur.*"

If this scenario was to occur (and surely there is nothing theologically suspect here for classical theism) an infinite temporal progression would be realised that is directly analogous to an infinite temporal regression; while the infinite *t*s of the past cannot be changed, re-ordered or removed, nor can infinite divinely predetermined future *t*s. Gabriel and Uriel's praise events can be placed in a one-to-one correspondence with the natural numbers, or the even and odd numbers, and further, in the case of an eternal past, all *tnow -n* and all *praisenow +n* can also be put in one-to-one correspondence.

Morriston makes his case very clearly. This scenario does not constitute a potential infinity. The number of praises is not actually growing; the amount of praises is fixed, having been predetermined by God to be an infinite set. God could change his mind at the last moment to prevent Uriel from saying any of his praises without having any effect on the infinite praises yet to be said. Similarly, instead of giving praise at every second celestial minute God could have Gabriel change his regime and, say, remain mute at every third praise after his first billion without altering the infinite nature of his set of praises. If the series is predetermined to be endless, regardless of how many praises are added or subtracted the result would remain an infinite set. As Cohen (2015) puts it:

"*Since these alleged absurdities are similar in kind to the alleged absurdities that follow from a beginningless past, it follows that Craig's argument against the possibility of a beginningless past results in an equally good argument against the possibility of an endless future, or at least a predetermined one.*"

A number of possible refutations have been attempted. Nevertheless, they fail, often with consequences unkind to theism. For example, it might be argued that we are unable to enumerate events that are non-existent because they haven't yet occurred, whereas we can enumerate past events because we know them to have occurred. This misses the point. What is being counted is not necessarily the praises themselves, i.e., not the kind of events *per se* but the *t*s; the 'equal intervals of time' (i.e., the temporal moments or "*one minute of celestial time*"). There is no reason why we cannot mentally number and sequentially count predetermined future *t*s in exactly the same manner as we can past *t*s, regardless of the kinds of events that occurred at any instance of *t*. Nor does it make any difference which arbitrary measures we choose to continuously count, so long as they are negative or positive integers from some temporal point designated as zero. We could just as easily use the time it takes to complete one praise event, one Earth orbit around our sun, or the decay rate of Plutonium-235.

Rebuttals have also been made by invoking the A-theory of time but, unsurprisingly, they also do not hold. Recall that the

presentist would claim that past *t*s and future *t*s have an ontological status not shared by the *tnow* or p*raisenow* moment because this is the only moment that actually exists. For example al-Ghazali (2000) wrote:

"*The future does not enter at all into existence, either successively or concomitantly, whereas all of the past has entered into existence successively, even though not concomitantly.*"

An argument echoed by Craig (2008):

".....events in time are not all equally real: the future does not yet exist and the past no longer exists; only things which are present are real."

It reasonably follows, should this be true, that there would be no actual future praises, only potential future praises, none of which can be given any quantifier such as p*raisenow* +1, p*raisenow* +2..... However, accepting this workaround entails that the claimant should also be unable to count or otherwise quantify an infinite amount of past praises (i.e., p*raisenow* -1, p*raisenow* -2.....) regressing back to any p*raisebeginning*, in the same way that they claim it is absurd to imagine counting back to infinity prior to *tnow*. Craig actually contradicts himself here because he has previously argued (1990) that a predetermined future is perfectly plausible within classical theism. And, as Morriston (2012) points out, if God decreed that Gabriel and Uriel were to say some finite number of praises then they would certainly be able to be enumerated beforehand even though a finite amount of praises would be just as non-existent as an infinite amount. This leads the presentist into a logical morass. If they wish to claim that past and future events do not exist they must also bite the bullet and accept that there can be no truths about past and future events, at least at *tnow*. Surely to the classical theist, though, a future moment or event predetermined by God must be true; it must be actual. It cannot, for example, merely have a very high probability of occurring. And any temporal moment or event that is actual must surely exist.

The problem is obvious: under presentism *t*s are veridical (becomes actualised, i.e., become *tnow*) in their own time and only in their own time. Therefore, within an infinite set of *t*s, presentism holds that there can never exist a time when all moments in that series simultaneously exist or are actualised (*per* the failure of a concurrently existing Hilbert's Hotel to model an infinite regression). It follows that there can never be any *t* when all possible *t*s in the series have been actualised. In other words, every *tnow+n* is no more than a non-existent, unactualised possible *t* and so an infinite set involving a predetermined temporal projection could never occur. Yet the manner in which Morriston's scenario is constructed means that under tense logic (Prior, 1967; 1969) the proposition: 'an infinite amount of praises will be said' is true:

(an infinite amount of praises will be said): is true
P (an infinite amount of praises will be said): is true

So for the presentist subscribing to classical theism, when confronted with Morriston's scenario, there are only two plausible outcomes: either presentism is false or the certainty of a predetermined future ordained by God is false. To keep presentism, the predetermined endless praises can never exist as an infinite set and so an endless future is impossible. And if a predetermined endless future is impossible, so is a non-determined endless future. This is surely not something a classical theist, for whom the concept of eternity is an absolute, would countenance.

An alternative strategy might be to consider past *t*s to possess some ontological status not shared by future *t*s. Perhaps past *t*s can be enumerated because they have been actualised at some *t* while future *t*s cannot be enumerated because they have yet to be actualised at any *t*. Thus future *t*s are never more than potential (no matter how certain we can be that they will occur) and cannot be included in what we might call reality. From Craig (2010b):

"*Everything that has happened has been actualized. As the medievals put it, these events have exited from their causes and are therefore no longer in potentiality. The actual world thus includes both what does exist and what did exist. But events which have yet to take place, being pure potentialities, are, on a tensed view of time, not part of the actual world . . . Even if past events do not exist, they are still part of the actual world in a way that future events are not, since the actual world comprises everything that has happened.*"

This view, that all *tnow-n* are somehow partially real, *tnow* is fully real and all *tnow+n* are non-real represents, in the case

of *tnow–n*, a distancing from the orthodox presentist stance, being more akin to a third notion of time, the 'growing block' or possibilist' theory (e.g., Broad, 1923; Tooley, 1997) which has even less advocates than the A-theory and presentism. This view finds itself perched mid-way between the B-theory (in that it states that both *tnow* and *tnow-n* are equally real) and the A-theory and presentism (in that it denies that all *tnow+n* are real). Thus, under possibilism what exists is claimed to be what has been actualised, bounded by a 'growing temporal edge' that continuously projects into the future. Broad (1923) explains:

"*Nothing has happened to the present by becoming past except that fresh slices of existence have been added to the total history of the world. The past is thus as real as the present*."

Craig appears to be only partially endorsing the possibilist view. He seems to be claiming here that future moments can neither exist, be actual or quantifiable while all *tnow-n* do not exist, yet are somehow deemed actual (and quantifiable) because they have existed at some *t*. However, even if Craig were fully endorsing the possibilist view as a means of sidestepping Morriston's scenario it would be ineffective because he hasn't escaped the temporal relational problems that plague presentism. If both *tnow–n* and *tnow* are equally existent it is easy enough to differentiate *tnow–n* and *tnow* from *tnow+n*, but how does one then differentiate *tnow* from *tnow –n*?

Merricks (2006) illustrates the problem in the following way. He asks us to imagine Emperor Nero sitting in the Coliseum watching gladiatorial combat and thinking something along the lines, "*I am sitting here at the present time in the Coliseum watching gladiatorial combat*." Now he invites the present reader to entertain a similar thought such as, "*I am sitting here at the present time reading this essay*." On the possibilist theory of time, both events are equally veridical from the time they have occurred and projecting into the future. To preserve the possibilist view, then, a distinction needs to be made between the objective *tnow* (the hypothesised temporal edge) and myriad subjective *tnow*s (in this example the two equally veridical *tnow*s for Nero and the reader of this essay). If this distinction cannot be achieved *tnow* (and the growing temporal edge) will remain undeterminable (Braddon-Mitchell, 2004; Merricks, 2006), leaving the reader of this essay with no right to assume that their subjective *tnow* and the objective *tnow* actually coincided. And, by extension, the possibilist also needs some means by which to distinguish the subjective *tnow+n* (e.g., the different tomorrows for Nero and the reader) from the objective *tnow+n (*e.g., tomorrow for the growing temporal edge). As Merricks (2006) contends, possibilism is:

"*.....like the desire to eat one's cake but also have it. It is understandable and incoherent*."

Finally, strange as it might sound, attempts have been made by Craig to counter Morriston's scenario by conflating an infinite set with zero. Here, Craig seems to be ditching formal mathematics entirely and taking his cue from the pseudomathematics of popular literature such as the aforementioned Seife (2000) who waxes lyrically that:

"*Zero and infinity always looked suspiciously alike.....it is possible to understand the infinite by studying zero.....most appalling of all, infinity can be a zero.....Nature has defended itself from an unruly zero.....zero and infinity are eternally locked in a struggle to engulf all the numbers.....However, zero is too powerful even for nature*."

Seife's rationale includes that he finds black holes to be an absurd consequence of General Relativity. Like Craig, he too finds absurdities where none exist (see Gray, 2000, for a non-technical review of Seife's mistaken notions of mathematics and physics). When discussing the singularity hypothesised to have existed at the beginning of the universe, Craig (2002), apparently agreeing with Seife, had this to say:

"*This event that marked the beginning of the universe becomes all the more amazing when one reflects on the fact that a state of 'infinite density' is synonymous to 'nothing'. There can be no object that possesses infinite density, for if it had any size at all it could still be even more dense*."

Here, Craig appears to be denying not only that infinity exists but neither do singularities, for a singularity is defined as an object that possesses infinite density. Morriston (2002) is, quite rightly, having none of this:

"*No-one would suppose that it follows from the fact that there can be no 'round squares' that 'round square' is synonymous with nothing. But neither should anyone suppose that it follows from the fact (assuming it is a fact) that there can be no infinitely dense objects, that 'infinite density is synonymous with 'nothing*.'"

Nevertheless, Craig (2010b) has attempted the same kind of rebuttal in response to Morriston's Praises scenario. In answer to the question "*What is the number of praises in the series of future praises*?" Craig answers "*none*". This is both absurd and contradictory. By "*none*" he surely cannot literally mean zero. This would imply that no praises will ever be said or could exist (despite God's command, again, an uneasy claim for a classical theist to make). Furthermore, if he does mean zero, then it follows, on his own reasoning, that an infinite temporal regression must be comprised of zero *t*s, which would preclude even *tnow* from existing.

Either way, Craig's refutation harbours another sombre problem for classical theism. Morriston (2012) points out that simply invoking the transitive property of equality (i.e., if $x = y$, then $y = x$) demonstrates that, if an infinite set equates to zero, then creation *ex nihilo* becomes a logical impossibility. This is because 'absolute nothingness' in the strict philosophical sense (a completely null set that is not even a set, being devoid of any physical, abstract, logical or metaphysical properties at all; *viz.* creation *ex nihilo*) would effectively equate to an infinite set. In keeping with Craig's insistence on treating infinity as a number, therefore, Craig's reference to "*none*" is possibly intended to imply a quantity so large in magnitude that it can only be described as not finite and so undetermined within number theory. In other words, an infinite set. Indeed, when the question is rephrased as, "*How many praises will be said*?" Craig answers differently, again ignoring mathematics and falling back on a dubious philosophical notion: "*potentially infinitely many*". Yet, as Morriston (2012) points out, the word "*potentially*" is redundant here, because the amount of praises said is not actually growing.

Conclusion

A variety of physical, philosophical and mathematical objections to the claim 'you cannot count back from infinity' have been presented. Specifically, to uphold the claim requires several suppositions to be true, including the assumptions that:

1. Special Relativity is flawed
2. The A-theory of time and especially 'presentism' (or perhaps 'possiblism') obtains; the B-theory does not
3. Time must exist in discrete, objectively measurable quanta
4. We can substitute the well-defined mathematical concept of an infinite set with the vague philosophical and mathematically undefined notion of potential infinity
5. We can ignore the consistent logical mathematical mechanisms for dealing with infinite sets and replace them with human intuitions pertaining to finite numbers
6. We should expect infinite sets to conform to arithmetic operations derived from the Peano axioms
7. We can selectively apply the claim to infinite temporal regressions while ignoring analogous problems for infinite temporal progressions

Together these assumptions constitute belief in at least six impossible things before breakfast. Mathematically alone, however, the claim fails as it is no more than coherent than this:

P1: If an actual infinity (or infinite temporal regress) exists, then
P2: We will not be able to perform standard arithmetic operations on this infinite set without producing absurd results
P3: We do receive absurd results when we use standard arithmetic operations with infinite sets
C: Therefore: the universe cannot have an infinite age.

This is a classic *non-sequitur*; in no way does the conclusion follow from the premises. For those who choose to persevere with the notion that the claim holds water Morriston (2013) points out their own absurdity particularly well:

"*.....it would be like arguing that there must be finitely many natural numbers because we can't stop counting them.*"

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